Factoring Quadratic Expressions

From Foundations to Advanced Factoring

ALGEBRA II

Factoring is the reverse of multiplication. The goal is to rewrite an expression as a product of simpler expressions.

Quick Check: Multiply First

Multiply first: $(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$

Factoring reverses this: $x^2 + 8x + 15 = (x + 3)(x + 5)$

How this worksheet progresses

FOUNDATION: GCF → DEVELOPING: $x^2 + bx + c$ → ON-LEVEL: $ax^2 + bx + c$ → SPECIAL PATTERNS: difference of squares → ADVANCED: mixed factoring → APPLICATION: solving by factoring → CHALLENGE: reasoning & error analysis

FOUNDATION Level 1: Greatest Common Factor

Strategy

Always check for a GCF before using any other factoring method.

Worked example: $6x^2 + 18x \rightarrow \text{GCF} = 6x \rightarrow 6x(x + 3)$

Check: multiply your factors back together.

Directions: Factor out the greatest common factor.

1. $8x + 20$

2. $9x - 15$

3. $12x^2 + 18x$

4. $10x^2 - 25x$

5. $15x^3 - 25x^2$

6. $24x^4 + 16x^3$

7. $14x^2y + 21xy^2$

8. $18a^3b - 12a^2b^2$

9. $7x^2 + 7x$

10. $20x^3 + 30x^2 - 50x$

11. $9m^4 - 6m^3 + 12m^2$

12. $22p^2q^3 + 33p^3q^2$

DEVELOPING

Level 2: Factor $x^2 + bx + c$

Strategy

For $x^2 + bx + c$, find two numbers that **multiply to c** and **add to b**.

Worked example: $x^2 + 7x + 12$. Two numbers multiply to 12 and add to 7: 3 and 4.

So $x^2 + 7x + 12 = (x + 3)(x + 4)$.

Directions: Factor each trinomial. For problems 1 and 8, first list the factor pairs of the constant term.

1. $x^2 + 9x + 20$

2. $x^2 + 8x + 15$

3. $x^2 + 10x + 21$

4. $x^2 + 13x + 36$

5. $x^2 - 9x + 20$

6. $x^2 - 11x + 24$

7. $x^2 - 12x + 35$

8. $x^2 + x - 12$

9. $x^2 - 2x - 15$

10. $x^2 + 5x - 24$

11. $x^2 - 4x - 32$

12. $x^2 + 14x + 45$

13. $x^2 - x - 42$

14. $x^2 + 3x - 54$

ON-LEVEL

Level 3: Factor $ax^2 + bx + c$ (AC Method)

AC / Decomposition Method

For $6x^2 + 11x + 3$: $a \times c = 6 \times 3 = 18$.

Find two numbers that multiply to 18 and add to 11: 9 and 2.

Rewrite: $6x^2 + 9x + 2x + 3$

Group: $3x(2x + 3) + 1(2x + 3)$

Factor: $(3x + 1)(2x + 3)$

Directions: Factor completely. Problems 10 and 11 require you to remove a GCF first.

1. $2x^2 + 7x + 3$

2. $3x^2 + 10x + 8$

3. $6x^2 + 11x + 3$

4. $5x^2 + 13x + 6$

5. $4x^2 - 8x + 3$

6. $6x^2 - 17x + 5$

7. $3x^2 - 2x - 8$

8. $8x^2 + 2x - 3$

9. $10x^2 - 9x - 9$

10. $12x^2 + 26x + 10$

11. $18x^2 - 21x - 9$

SPECIAL PATTERNS

Level 4: Difference of Squares

Pattern

$a^2 - b^2 = (a - b)(a + b)$

$x^2 - 25 = x^2 - 5^2 = (x - 5)(x + 5)$

$9x^2 - 16 = (3x)^2 - 4^2 = (3x - 4)(3x + 4)$

With a GCF first: $2x^2 - 50 = 2(x^2 - 25) = 2(x - 5)(x + 5)$

Directions: Factor completely.

1. $x^2 - 25$

2. $x^2 - 81$

3. $9x^2 - 16$

4. $25x^2 - 49$

5. $4x^2 - 121$

6. $100 - x^2$

7. $2x^2 - 50$

8. $3x^2 - 27$

9. $16x^4 - 81$

10. Can $x^2 + 36$ be factored using the difference-of-squares pattern? Explain.

ADVANCED

Level 5: Choose the Method

Ask yourself

Is there a GCF?
How many terms are there?
Is it a difference of squares?
Is it a trinomial?
Can it be factored further?

Directions: Factor each expression completely. First decide which method applies. If an expression cannot be factored with integer coefficients, write **Prime**.

1. $5x^2 - 20$

2. $x^2 + 7x + 10$

3. $3x^2 + 12x$

4. $x^2 - 36$

5. $2x^2 + 7x + 6$

6. $x^2 + 4x + 7$

7. $4x^2 - 36$

8. $x^2 - 5x - 14$

9. $6x^2 + x - 2$

10. $2x^2 + 8x + 6$

11. $x^2 + 9$

12. $9x^2 - 30x + 25$

13. $x^3 - 4x$

14. $x^2 + 2x - 4$

APPLICATION

Level 6: Solve Quadratic Equations by Factoring

Strategy

Zero Product Property: if $AB = 0$, then $A = 0$ or $B = 0$.

Worked example: $x^2 + 5x + 6 = 0 \rightarrow (x + 2)(x + 3) = 0$

$x + 2 = 0 \rightarrow x = -2$ and $x + 3 = 0 \rightarrow x = -3$

Answer: $x = -2, -3$

Directions: Solve each equation by factoring. Move all terms to one side first when needed.

1. $x^2 + 5x + 6 = 0$

2. $x^2 - 7x + 12 = 0$

3. $x^2 + x - 20 = 0$

4. $x^2 = 49$

5. $x^2 - 10x = 0$

6. $2x^2 + 5x + 3 = 0$

7. $3x^2 - 10x + 8 = 0$

8. $x^2 + 6x = -8$

9. $4x^2 - 9 = 0$

10. $2x^2 - 8x = 0$

CHALLENGE

Advanced Factoring Challenge

1. Multi-step factoring. Factor completely: $3x^4 - 48x^2$.

2. Find the missing value. $x^2 + kx + 24$ factors as $(x + 4)(x + 6)$. Find k.

3. Find the missing value. $x^2 - 13x + c$ factors as $(x - 4)(x - 9)$. Find c.

4. Error analysis. A student writes $x^2 - 16 = (x - 4)^2$. What mistake was made? What is the correct factorization? How can multiplication verify the answer?

5. Create your own. Write the quadratic expression whose factors are $(2x - 3)(x + 5)$. Expand it and explain how you know your factorization is correct.

6. Reasoning. Why can't $x^2 + 16$ be factored as $(x + 4)(x - 4)$? Explain in words.

7. Solve & verify. Solve $2x^2 - 5x - 3 = 0$ by factoring, then substitute each solution back into the original equation to verify.

ANSWER KEY

Answer Key

Level 1 — GCF

1. $4(2x + 5)$
2. $3(3x - 5)$
3. $6x(2x + 3)$
4. $5x(2x - 5)$
5. $5x^2(3x - 5)$
6. $8x^3(3x + 2)$
7. $7xy(2x + 3y)$
8. $6a^2b(3a - 2b)$
9. $7x(x + 1)$
10. $10x(2x^2 + 3x - 5)$
11. $3m^2(3m^2 - 2m + 4)$
12. $11p^2q^2(2q + 3p)$

Level 2 — $x^2 + bx + c$

1. $(x + 4)(x + 5)$ [$4 \times 5 = 20$, $4 + 5 = 9$]
2. $(x + 3)(x + 5)$
3. $(x + 3)(x + 7)$
4. $(x + 4)(x + 9)$
5. $(x - 4)(x - 5)$
6. $(x - 3)(x - 8)$
7. $(x - 5)(x - 7)$
8. $(x + 4)(x - 3)$ [$4 \times (-3) = -12$, $4 - 3 = 1$]
9. $(x - 5)(x + 3)$
10. $(x + 8)(x - 3)$
11. $(x - 8)(x + 4)$
12. $(x + 5)(x + 9)$
13. $(x - 7)(x + 6)$
14. $(x + 9)(x - 6)$

Level 3 — $ax^2 + bx + c$

1. $(2x + 1)(x + 3)$
2. $(3x + 4)(x + 2)$
3. $ac = 18$; $9 + 2 = 11 \rightarrow 6x^2 + 9x + 2x + 3 = 3x(2x + 3) + 1(2x + 3) =$ **$(3x + 1)(2x + 3)$**
4. $(5x + 3)(x + 2)$
5. $(2x - 1)(2x - 3)$
6. $ac = 30$; $-15 - 2 = -17 \rightarrow 6x^2 - 15x - 2x + 5 = 3x(2x - 5) - 1(2x - 5) =$ **$(3x - 1)(2x - 5)$**
7. $(3x + 4)(x - 2)$
8. $(4x + 3)(2x - 1)$
9. $ac = -90$; $-15 + 6 = -9 \rightarrow 10x^2 - 15x + 6x - 9 = 5x(2x - 3) + 3(2x - 3) =$ **$(5x + 3)(2x - 3)$**
10. GCF 2 $\rightarrow 2(6x^2 + 13x + 5) =$ **$2(3x + 5)(2x + 1)$**
11. GCF 3 $\rightarrow 3(6x^2 - 7x - 3) =$ **$3(3x + 1)(2x - 3)$**

ANSWER KEY

Answer Key (continued)

Level 4 — Difference of Squares

1. $(x - 5)(x + 5)$
2. $(x - 9)(x + 9)$
3. $(3x - 4)(3x + 4)$
4. $(5x - 7)(5x + 7)$
5. $(2x - 11)(2x + 11)$
6. $(10 - x)(10 + x)$
7. $2(x - 5)(x + 5)$
8. $3(x - 3)(x + 3)$
9. $(4x^2 - 9)(4x^2 + 9) = (2x - 3)(2x + 3)(4x^2 + 9)$
10. No. $x^2 + 36$ is a *sum* of squares, not a difference, so the pattern does not apply. It is prime over the integers.

Level 5 — Mixed Factoring

1. $5(x - 2)(x + 2)$
2. $(x + 2)(x + 5)$
3. $3x(x + 4)$
4. $(x - 6)(x + 6)$
5. $(2x + 3)(x + 2)$
6. Prime
7. $4(x - 3)(x + 3)$
8. $(x - 7)(x + 2)$
9. $(3x + 2)(2x - 1)$
10. $2(x + 1)(x + 3)$
11. Prime
12. $(3x - 5)^2$
13. $x(x - 2)(x + 2)$
14. Prime

Level 6 — Solve by Factoring

1. $(x + 2)(x + 3) = 0 \rightarrow x = -2, -3$
2. $(x - 3)(x - 4) = 0 \rightarrow x = 3, 4$
3. $(x + 5)(x - 4) = 0 \rightarrow x = -5, 4$
4. $x^2 - 49 = 0 \rightarrow (x - 7)(x + 7) = 0 \rightarrow x = 7, -7$
5. $x(x - 10) = 0 \rightarrow x = 0, 10$
6. $(2x + 3)(x + 1) = 0 \rightarrow x = -3/2, -1$
7. $(3x - 4)(x - 2) = 0 \rightarrow x = 4/3, 2$
8. $x^2 + 6x + 8 = 0 \rightarrow (x + 2)(x + 4) = 0 \rightarrow x = -2, -4$
9. $(2x - 3)(2x + 3) = 0 \rightarrow x = 3/2, -3/2$
10. $2x(x - 4) = 0 \rightarrow x = 0, 4$

Challenge

1. GCF $3x^2 \rightarrow 3x^2(x^2 - 16) =$ $\mathbf{3x^2(x - 4)(x + 4)}$
2. $(x + 4)(x + 6) = x^2 + 10x + 24$, so $\mathbf{k = 10}$.
3. $(x - 4)(x - 9) = x^2 - 13x + 36$, so $\mathbf{c = 36}$.
4. The student used a perfect-square pattern on a difference of squares. $(x - 4)^2 = x^2 - 8x + 16$, which has a middle term. Correct: $\mathbf{x^2 - 16 = (x - 4)(x + 4)}$; multiplying gives $x^2 + 4x - 4x - 16 = x^2 - 16$. ✓
5. $(2x - 3)(x + 5) = 2x^2 + 10x - 3x - 15 =$ $\mathbf{2x^2 + 7x - 15}$. Expanding returns the original expression, which verifies the factorization.
6. $(x + 4)(x - 4)$ expands to $x^2 - 16$, not $x^2 + 16$. A sum of squares has no integer factorization, so $x^2 + 16$ is prime over the integers.
7. $2x^2 - 5x - 3 = (2x + 1)(x - 3) = 0 \rightarrow$ $\mathbf{x = -1/2, 3}$. Check $x = 3$: $2(9) - 15 - 3 = 0$ ✓. Check $x = -1/2$: $2(1/4) + 5/2 - 3 = 0.5 + 2.5 - 3 = 0$ ✓